Formulas

Formulas from Coordinate Geometry

Slope of a line (p. 82)	$m = \frac{y_2 - y_1}{x_2 - x_1}$ where m is the slope of the nonvertical line through points (x_1, y_1) and (x_2, y_2)
Parallel and perpendicular lines (p. 84)	If line l_1 has slope m_1 and line l_2 has slope m_2, then: $l_1 \parallel l_2$ if and only if $m_1 = m_2$; $l_1 \perp l_2$ if and only if $m_1 = -\frac{1}{m_2}$, or $m_1 m_2 = -1$
Distance formula (p. 615)	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ where d is the distance between points (x_1, y_1) and (x_2, y_2)
Midpoint formula (p. 615)	$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is the midpoint of the line segment joining points (x_1, y_1) and (x_2, y_2).

Formulas from Matrix Algebra

Determinant of a 2 × 2 matrix (p. 203)	$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$
Determinant of a 3 × 3 matrix (p. 203)	$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb)$
Area of a triangle (p. 204)	The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by $\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ where the appropriate sign (±) should be chosen to yield a positive value.
Cramer's rule (p. 205)	Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be the coefficient matrix of this linear system: $ax + by = e$, $cx + dy = f$. If $\det A \neq 0$, then the system has exactly one solution. The solution is $x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A}$ and $y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$.
Inverse of a 2 × 2 matrix (p. 210)	The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{\lvert A \rvert}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ provided $ad - cb \neq 0$.

Formulas and Theorems from Algebra

Quadratic formula (p. 292)	The solutions of $ax^2 + bx + c = 0$ are $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ where a, b, and c are real numbers such that $a \neq 0$.
Discriminant of a quadratic equation (p. 294)	The expression $b^2 - 4ac$ is called the discriminant of the associated equation $ax^2 + bx + c = 0$. The value of the discriminant can be positive, zero, or negative, which corresponds to an equation having two real solutions, one real solution, or two imaginary solutions, respectively.
Special product patterns (p. 347)	**Sum and difference:** $(a + b)(a - b) = a^2 - b^2$ **Square of a binomial:** $(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2ab + b^2$ **Cube of a binomial:** $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
Special factoring patterns (p. 354)	**Sum of two cubes:** $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ **Difference of two cubes:** $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
Remainder theorem (p. 363)	If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.
Factor theorem (p. 364)	A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.
Rational zero theorem (p. 370)	If $f(x) = a_nx^n + \cdots + a_1x + a_0$ has *integer* coefficients, then every rational zero of f has this form: $$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$
Fundamental theorem of algebra (p. 379)	If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has at least one solution in the set of complex numbers.
Corollary to the fundamental theorem of algebra (p. 379)	If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has exactly n solutions provided each solution repeated twice is counted as 2 solutions, each solution repeated three times is counted as 3 solutions, and so on.
Complex conjugates theorem (p. 380)	If f is a polynomial function with real coefficients, and $a + bi$ is an imaginary zero of f, then $a - bi$ is also a zero of f.
Irrational conjugates theorem (p. 380)	Suppose f is a polynomial function with rational coefficients, and a and b are rational numbers such that $\sqrt{b}$ is irrational. If $a + \sqrt{b}$ is a zero of f, then $a - \sqrt{b}$ is also a zero of f.
Descartes' rule of signs (p. 381)	Let $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ be a polynomial function with real coefficients. • The number of *positive real zeros* of f is equal to the number of changes in sign of the coefficients of $f(x)$ or is less than this by an even number. • The number of *negative real zeros* of f is equal to the number of changes in sign of the coefficients of $f(-x)$ or is less than this by an even number.

Formulas and Theorems from Algebra *(continued)*

Discriminant of a general second-degree equation (p. 653)	Any conic can be described by a general second-degree equation in x and y: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. The expression $B^2 - 4AC$ is the discriminant of the conic equation and can be used to identify it. **Discriminant** — **Type of Conic** $B^2 - 4AC < 0$, $B = 0$, and $A = C$ — Circle $B^2 - 4AC < 0$, and either $B \neq 0$ or $A \neq C$ — Ellipse $B^2 - 4AC = 0$ — Parabola $B^2 - 4AC > 0$ — Hyperbola If $B = 0$, each axis of the conic is horizontal or vertical.

Formulas from Combinatorics

Fundamental counting principle (p. 682)	If one event can occur in m ways and another event can occur in n ways, then the number of ways that both events can occur is $m \cdot n$.
Permutations of *n* objects taken *r* at a time (p. 685)	The number of permutations of r objects taken from a group of n distinct objects is denoted by ${}_nP_r$ and is given by: $${}_nP_r = \frac{n!}{(n-r)!}$$
Permutations with repetition (p. 685)	The number of distinguishable permutations of n objects where one object is repeated s_1 times, another is repeated s_2 times, and so on is: $$\frac{n!}{s_1! \cdot s_2! \cdot \ldots \cdot s_k!}$$
Combinations of *n* objects taken *r* at a time (p. 690)	The number of combinations of r objects taken from a group of n distinct objects is denoted by ${}_nC_r$ and is given by: $${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$$
Pascal's triangle (p. 692)	If you arrange the values of ${}_nC_r$ in a triangular pattern in which each row corresponds to a value of n, you get what is called Pascal's triangle. ${}_0C_0$ — 1 ${}_1C_0 \quad {}_1C_1$ — 1 1 ${}_2C_0 \quad {}_2C_1 \quad {}_2C_2$ — 1 2 1 ${}_3C_0 \quad {}_3C_1 \quad {}_3C_2 \quad {}_3C_3$ — 1 3 3 1 ${}_4C_0 \quad {}_4C_1 \quad {}_4C_2 \quad {}_4C_3 \quad {}_4C_4$ — 1 4 6 4 1 The first and last numbers in each row are 1. Every number other than 1 is the sum of the closest two numbers in the row directly above it.
Binomial theorem (p. 693)	The binomial expansion of $(a + b)^n$ for any positive integer n is: $$(a+b)^n = {}_nC_0 a^n b^0 + {}_nC_1 a^{n-1} b^1 + {}_nC_2 a^{n-2} b^2 + \cdots + {}_nC_n a^0 b^n = \sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$$

TABLES

Formulas from Probability

Theoretical probability of an event (p. 698)	When all outcomes are equally likely, the theoretical probability that an event A will occur is: $P(A) = \frac{\text{Number of outcomes in } A}{\text{Total number of outcomes}}$
Odds in favor of an event (p. 699)	When all outcomes are equally likely, the odds in favor of an event A are: $\frac{\text{Number of outcomes in } A}{\text{Number of outcomes not in } A}$
Odds against an event (p. 699)	When all outcomes are equally likely, the odds against an event A are: $\frac{\text{Number of outcomes not in } A}{\text{Number of outcomes in } A}$
Experimental probability of an event (p. 700)	When an experiment is performed that consists of a certain number of trials, the experimental probability of an event A is given by: $P(A) = \frac{\text{Number of trials where } A \text{ occurs}}{\text{Total number of trials}}$
Probability of compound events (p. 707)	If A and B are any two events, then the probability of A or B is: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ If A and B are disjoint events, then the probability of A or B is: $P(A \text{ or } B) = P(A) + P(B)$
Probability of the complement of an event (p. 709)	The probability of the complement of event A, denoted $\overline{A}$, is: $P(\overline{A}) = 1 - P(A)$
Probability of independent events (p. 717)	If A and B are independent, the probability that both A and B occur is: $P(A \text{ and } B) = P(A) \cdot P(B)$
Probability of dependent events (p. 718)	If A and B are dependent, the probability that both A and B occur is: $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$
Binomial probabilities (p. 725)	For a binomial experiment consisting of n trials where the probability of success on each trial is p, the probability of exactly k successes is: $P(k \text{ successes}) = {}_nC_k p^k (1 - p)^{n-k}$

Formulas from Statistics

Mean of a data set (p. 744)	$\overline{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$ where $\overline{x}$ (read "x-bar") is the mean of the data $x_1, x_2, \ldots, x_n$
Standard deviation of a data set (p. 745)	$\sigma = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \ldots + (x_n - \overline{x})^2}{n}}$ where σ (read "sigma") is the standard deviation of the data $x_1, x_2, \ldots, x_n$
Areas under a normal curve (p. 757)	A normal distribution with mean $\overline{x}$ and standard deviation σ has these properties: • The total area under the related normal curve is 1. • About 68% of the area lies within 1 standard deviation of the mean. • About 95% of the area lies within 2 standard deviations of the mean. • About 99.7% of the area lies within 3 standard deviations of the mean.
***z*-score (p. 758)**	$z = \frac{x - \overline{x}}{\sigma}$ where x is a data value, $\overline{x}$ is the mean, and σ is the standard deviation

Formulas for Sequences and Series

Formulas for sums of special series **(p. 797)**	$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
Explicit rule for an arithmetic sequence **(p. 802)**	The nth term of an arithmetic sequence with first term a_1 and common difference d is: $$a_n = a_1 + (n-1)d$$
Sum of a finite arithmetic series **(p. 804)**	The sum of the first n terms of an arithmetic series is: $$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$
Explicit rule for a geometric sequence **(p. 810)**	The nth term of a geometric sequence with first term a_1 and common ratio r is: $$a_n = a_1 r^{n-1}$$
Sum of a finite geometric series **(p. 812)**	The sum of the first n terms of a geometric series with common ratio $r \neq 1$ is: $$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$
Sum of an infinite geometric series **(p. 821)**	The sum of an infinite geometric series with first term a_1 and common ratio r is $$S = \frac{a_1}{1-r}$$ provided $\lvert r \rvert < 1$. If $\lvert r \rvert \geq 1$, the series has no sum.
Recursive equation for an arithmetic sequence **(p. 827)**	$a_n = a_{n-1} + d$ where d is the common difference
Recursive equation for a geometric sequence **(p. 827)**	$a_n = r \cdot a_{n-1}$ where r is the common ratio

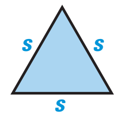
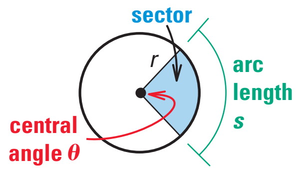
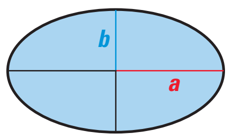
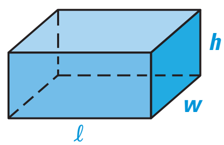
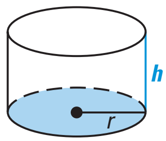
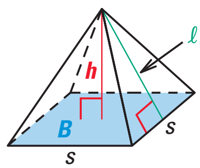
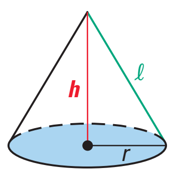
Formulas and Identities from Trigonometry

Conversion between degrees and radians **(p. 860)**	To rewrite a degree measure in radians, multiply by $\frac{\pi \text{ radians}}{180°}$. To rewrite a radian measure in degrees, multiply by $\frac{180°}{\pi \text{ radians}}$.
Definition of trigonometric functions **(p. 866)**	Let θ be an angle in standard position and (x, y) be any point (except the origin) on the terminal side of θ. Let $r = \sqrt{x^2 + y^2}$. $\sin \theta = \frac{y}{r}$ $\quad \cos \theta = \frac{x}{r}$ $\quad \tan \theta = \frac{y}{x}, x \neq 0$ $\csc \theta = \frac{r}{y}, y \neq 0$ $\quad \sec \theta = \frac{r}{x}, x \neq 0$ $\quad \cot \theta = \frac{x}{y}, y \neq 0$
Law of sines **(p. 882)**	If $\triangle ABC$ has sides of length a, b, and c, then: $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$
Area of a triangle (given two sides and the included angle) **(p. 885)**	If $\triangle ABC$ has sides of length a, b, and c, then its area is: $\text{Area} = \frac{1}{2}bc \sin A$ $\quad \text{Area} = \frac{1}{2}ac \sin B$ $\quad \text{Area} = \frac{1}{2}ab \sin C$

Formulas and Identities from Trigonometry *(continued)*

Law of cosines (p. 889)	If $\triangle ABC$ has sides of length a, b, and c, then: $$a^2 = b^2 + c^2 - 2bc \cos A$$ $$b^2 = a^2 + c^2 - 2ac \cos B$$ $$c^2 = a^2 + b^2 - 2ab \cos C$$
Heron's area formula (p. 891)	The area of the triangle with sides of length a, b, and c is $$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$ where $s = \frac{1}{2}(a + b + c)$.
Reciprocal identities (p. 924)	$\csc \theta = \frac{1}{\sin \theta}$ $\quad$ $\sec \theta = \frac{1}{\cos \theta}$ $\quad$ $\cot \theta = \frac{1}{\tan \theta}$
Tangent and cotangent identities (p. 924)	$\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\quad$ $\cot \theta = \frac{\cos \theta}{\sin \theta}$
Pythagorean identities (p. 924)	$\sin^2 \theta + \cos^2 \theta = 1$ $\quad$ $1 + \tan^2 \theta = \sec^2 \theta$ $\quad$ $1 + \cot^2 \theta = \csc^2 \theta$
Cofunction identities (p. 924)	$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$ $\quad$ $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$ $\quad$ $\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$
Negative angle identities (p. 924)	$\sin(-\theta) = -\sin \theta$ $\quad$ $\cos(-\theta) = \cos \theta$ $\quad$ $\tan(-\theta) = -\tan \theta$
Sum formulas (p. 949)	$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$ $$\cos(a + b) = \cos a \cos b - \sin a \sin b$$ $$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$
Difference formulas (p. 949)	$$\sin(a - b) = \sin a \cos b - \cos a \sin b$$ $$\cos(a - b) = \cos a \cos b + \sin a \sin b$$ $$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$
Double-angle formulas (p. 955)	$\cos 2a = \cos^2 a - \sin^2 a$ $\quad$ $\sin 2a = 2 \sin a \cos a$ $\cos 2a = 2\cos^2 a - 1$ $\quad$ $\tan 2a = \frac{2 \tan a}{1 - \tan^2 a}$ $\cos 2a = 1 - 2\sin^2 a$
Half-angle formulas (p. 955)	$\sin \frac{a}{2} = \pm\sqrt{\frac{1 - \cos a}{2}}$ $\quad$ $\tan \frac{a}{2} = \frac{1 - \cos a}{\sin a}$ $\cos \frac{a}{2} = \pm\sqrt{\frac{1 + \cos a}{2}}$ $\quad$ $\tan \frac{a}{2} = \frac{\sin a}{1 + \cos a}$ The signs of $\sin \frac{a}{2}$ and $\cos \frac{a}{2}$ depend on the quadrant in which $\frac{a}{2}$ lies.

Formulas from Geometry

Basic geometric figures	See pages 991–993 for area formulas for basic two-dimensional geometric figures.
Area of an equilateral triangle	Area $= \frac{\sqrt{3}}{4}s^2$ where s is the length of a side 
Arc length and area of a sector	Arc length $= r\theta$ where r is the radius and θ is the radian measure of the central angle that intercepts the arc Area $= \frac{1}{2}r^2\theta$ 
Area of an ellipse	Area $= \pi ab$ where a and b are half the lengths of the major and minor axes of the ellipse 
Volume and surface area of a right rectangular prism	Volume $= \ell wh$ where ℓ is the length, w is the width, and h is the height Surface area $= 2(\ell w + wh + \ell h)$ 
Volume and surface area of a right cylinder	Volume $= \pi r^2 h$ where r is the base radius and h is the height Lateral surface area $= 2\pi rh$ Surface area $= 2\pi r^2 + 2\pi rh$ 
Volume and surface area of a right regular pyramid	Volume $= \frac{1}{3}Bh$ where B is the area of the base and h is the height Lateral surface area $= \frac{1}{2}ns\ell$ where n is the number of sides of the base, s is the length of a side of the base, and ℓ is the slant height Surface area $= B + \frac{1}{2}ns\ell$ 
Volume and surface area of a right circular cone	Volume $= \frac{1}{3}\pi r^2 h$ where r is the base radius and h is the height Lateral surface area $= \pi r\ell$ where ℓ is the slant height Surface area $= \pi r^2 + \pi r\ell$ 
Volume and surface area of a sphere	Volume $= \frac{4}{3}\pi r^3$ where r is the radius Surface area $= 4\pi r^2$